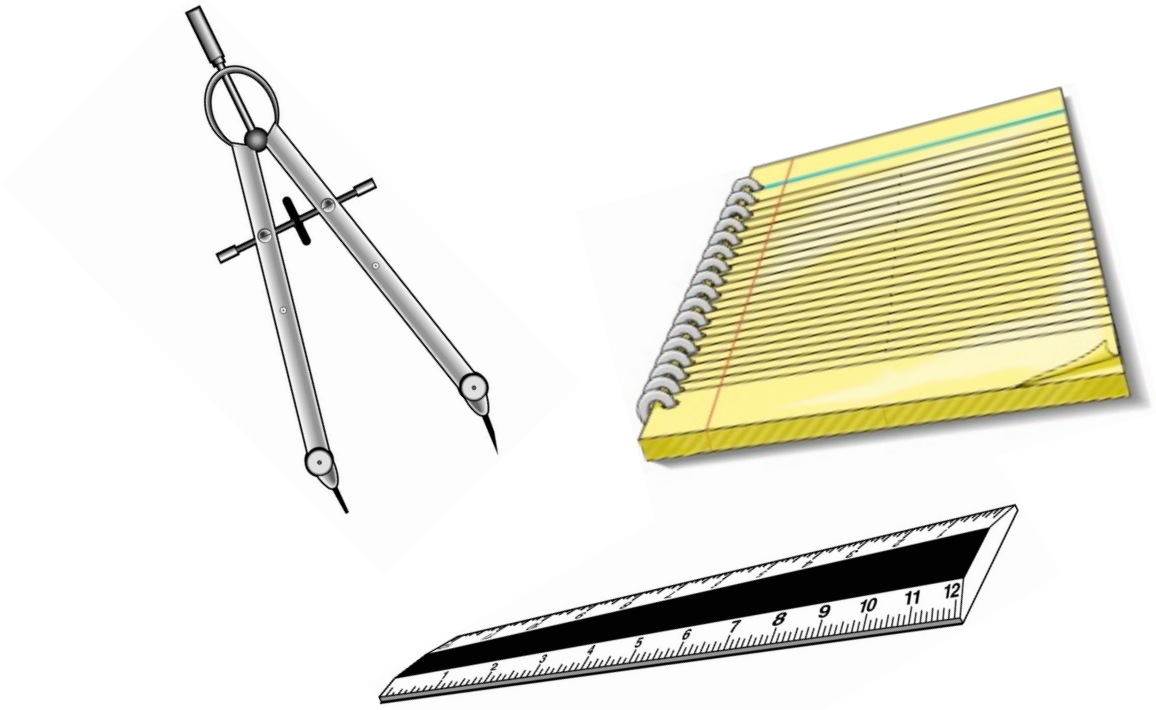
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

SAC • 2024



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YOU ARE INSTRUCTED TO DO SO!

1. Jack and Larry had supper at Bigham's Barbeque Friday night. Jack ordered the one-meat plate for $16.75, a slice of chocolate cake for $4.15, and an iced tea for $2.59. Larry ordered a two-meat plate for $18.95 and an iced tea for $2.59. The tax rate was 8.25%. Jack was feeling generous so he paid with three $20 bills and told the waitress to keep the change as a tip. How much was the tip?

(A) $11.14 **(B) $11.18** **(C) $11.22** **(D) $11.26** **(E) $11.30**

2. The Wylie math team held a fundraiser for their UIL team. They flew in the 60s rock group, the Ohio Express, and the concert was a sell-out. Adult tickets were priced at $22.75 and student tickets were priced at $14.50. They sold 2500 tickets and netted $48,501.25 How many adult tickets did they sell?

(A) 1485 **(B) 1488** **(C) 1491** **(D) 1494** **(E) 1497**

3. If $f(x) = \sqrt{x^3 + 22}$ and $h(x) = \ln(x) + 6$, then $f\big(h(55)\big) =$ _____. (nearest tenth)

(A) 31.8 **(B) 32.0** **(C) 32.2** **(D) 32.4** **(E) 32.6**

4. All of the houses on 6th street are the same size. Brennen can paint a house on 6th street by himself in 15 hours. If Luke works with him, they can paint a house on 6th street in 8 hr 45 min. How long does it take Luke to paint a house on 6th street by himself? (nearest whole number)

(A) 17 hr **(B) 18 hr** **(C) 19 hr** **(D) 20 hr** **(E) 21 hr**

5. The y-intercept of the line that contains the points $(-6, 4)$ and $(12, -2)$ is the point $(0, b)$.
b = _____. (nearest tenth)

(A) 1.6 **(B) 1.8** **(C) 2.0** **(D) 2.2** **(E) 2.4**

6. Find the domain of the function $f(x) = \dfrac{x-5}{\sqrt{9-x}}$.

(A) $x \in R,\ x \neq -5$ **(B) $x \in R,\ x \leq 9$** **(C) $x \in R,\ x > 9$** **(D) $x \in R,\ x < 9$** **(E) $x \in R,\ x \neq 5$**

7. The sound level of a sound is given by $\beta = 10\log\left(\dfrac{I}{I_0}\right)$, where β is the sound level in dB,
I is the intensity in W/m^2, and I_0 is the threshold of hearing which equals 10^{-12} W/m^2. If the sound level is 98 dB, then the intensity is _____ W/m^2. (nearest ten-thousandth)

(A) 0.0063 **(B) 0.0074** **(C) 0.0085** **(D) 0.0096** **(E) 0.0107**

8. If a 56-ft-tall tree produces a shadow that is 12 ft long, how long will the shadow be for a person that is 5 ft tall? (nearest hundredth)

(A) 1.07 ft **(B) 1.09 ft** **(C) 1.11 ft** **(D) 1.13 ft** **(E) 1.15 ft**

9-10. Consider a line containing points $A(-5,-1)$, $B(5,9)$, and $C(d,12)$.

9. The value of d is _____. (nearest tenth)

(A) 6 **(B) 7** **(C) 8** **(D) 9** **(E) 10**

10. If the point $F(e,3)$ lies on the perpendicular bisector of $\overline{AB}$, then e = _____.

(A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) 5**

11-14. Consider triangle ABC with vertices $A(2,8)$, $B(6,-2)$, and $C(-4,-4)$.

11. Find the perimeter of triangle ABC. (nearest tenth)

(A) 34.2 **(B) 34.4** **(C) 34.6** **(D) 34.8** **(E) 35.0**

12. The measure of $\angle ABC$ is _____°. (nearest tenth)

(A) 78.7 **(B) 78.9** **(C) 79.1** **(D) 79.3** **(E) 79.5**

13. The area of triangle ABC is _____. (nearest whole number)

(A) 50 **(B) 52** **(C) 54** **(D) 56** **(E) 58**

14. Given: Triangle ABC is similar to triangle DEF. If EF = 6.9, then DF = _____. (nearest tenth)

(A) 7.9 **(B) 8.3** **(C) 8.7** **(D) 9.1** **(E) 9.4**

15-17. Given: Circle with center O, diameter $\overline{CD}$, chord $\overline{EF}$ parallel to $\overline{CD}$. CD = 20 and EF = 16. $m\angle COE < 90°$.

15. If H is the midpoint of $\overline{EF}$, then OH = _____. (nearest tenth)

(A) 6.0 **(B) 6.2** **(C) 6.4** **(D) 6.6** **(E) 6.8**

16. The area of sector COE is _____. (nearest tenth)

(A) 32.0 **(B) 32.2** **(C) 32.4** **(D) 32.6** **(E) 32.8**

17. The arclength of minor arc EF is _____. (nearest tenth)

(A) 17.7 **(B) 17.9** **(C) 18.1** **(D) 18.3** **(E) 18.5**

18. A right circular cylinder has a diameter of 22 and a volume of 10,264. The total surface area of the cylinder is _____. (nearest whole number)

(A) 2610 **(B) 2614** **(C) 2618** **(D) 2622** **(E) 2626**

19-20. Consider isosceles trapezoid PQRS with $PQ = RS = 13$. $\overline{QR}$ is parallel to $\overline{PS}$. $QR = 16$ and $PS = 26$.

19. The area of trapezoid PQRS is _____. (nearest whole number)

(A) 250 (B) 252 (C) 254 (D) 256 (E) 258

20. PR = _____. (nearest tenth)

(A) 23.8 (B) 24.0 (C) 24.2 (D) 24.4 (E) 24.6

21. Consider the function $f(x) = 3x^3 + bx^2 - 21x - 30$. If $f(-2) = 36$, then b = _____.

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

22. The graph of $f(x) = \dfrac{x^2 - 16}{x^3 + x^2 - 12x}$ has _____ asymptotes.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. The graph of the circle $x^2 + y^2 = 49$ and the graph of the line $y = 0.6x + 5$ intersect at points A and B. AB = _____. (nearest tenth)

(A) 10.2 (B) 10.5 (C) 10.8 (D) 11.1 (E) 11.4

24. The graph of $y = 3\tan(.25x)$ has a vertical asymptote at x = _____.

(A) π (B) 2π (C) 3π (D) 4π (E) 5π

25. The diagonal of a television screen measures 54.12 inches. The width of the rectangularly shaped television screen is 23 inches greater than the height. The area of the television screen is _____ in^2. (nearest whole number)

(A) 1196 (B) 1200 (C) 1204 (D) 1208 (E) 1212

26. Consider the sequence 4, 11, 18, 25, 32, 39,… Find the sum of the first 14 terms.

(A) 693 (B) 695 (C) 697 (D) 699 (E) 701

27. Consider the sequence 40, 32, $\dfrac{128}{5}$, $\dfrac{512}{25}$,… Find the sum of the first 10 terms. (nearest tenth)

(A) 178.5 (B) 178.8 (C) 179.1 (D) 179.4 (E) 179.7

28. A hawk is perched at the edge of the roof of the Denver City State Bank. The hawk spots a mouse on the ground below. The angle of depression from the hawk to the mouse is 22°. The mouse is located 154 feet from the base of the bank. How tall is the Denver City State bank? (nearest foot)

(A) 48 ft **(B) 50 ft** **(C) 60 ft** **(D) 62 ft** **(E) 64 ft**

29. On March 1st of 2015, Piyush's father placed $75,000 into an account for Piyush that earns interest at a rate of 6.75% compounded quarterly. Piyush plans to withdraw all of the money in the account on March 1st of 2025 and use it toward the purchase of a new BMW X7 from Grapevine BMW. If the total cost including tax, title and license is $146,875.19 how much money will Piyush have to come up with? (nearest dollar)

(A) $300 **(B) $400** **(C) $500** **(D) $600** **(E) $700**

30. Consider the circle $x^2 + y^2 + ax + by + c = 0$. The center of the circle is the point $(2, 5)$ and the diameter is 14. $a + b + c =$ _____.

(A) –40 **(B) –38** **(C) –36** **(D) –34** **(E) –32**

31. A population of Fire Ants is increasing exponentially in Hale County. Phoenix introduced a population of 150 ants at t = 0. At t = 60 days, the population reached 1800 ants. The population should reach 212,000 ants at t = _____ days. (nearest whole number)

(A) 163 **(B) 166** **(C) 169** **(D) 172** **(E) 175**

32. Austin leaves the Lubbock Airport at 2:00 PM and flies on a bearing of 60° at a speed of 180 mph. At 2:30 PM, Zhikai leaves the Lubbock airport and flies on a bearing of 195° at a speed of 160 mph. How far apart are they at 4:00 PM? (nearest mile)

(A) 547 mi **(B) 550 mi** **(C) 553 mi** **(D) 556 mi** **(E) 559 mi**

33. Consider an ellipse centered at $(4, 3)$ with a vertex at $(–2, 3)$. The point $(4, 7)$ lies on the ellipse. The area of the ellipse is _____. (nearest tenth)

(A) 74.8 **(B) 75.1** **(C) 75.4** **(D) 75.7** **(E) 76.0**

34. Consider the curve represented by the parametric equations $x = 6\cos(\theta)$ and $y = 4\sin(\theta)$. The distance between the foci is _____. (nearest tenth)

(A) 8.3 **(B) 8.5** **(C) 8.7** **(D) 8.9** **(E) 9.1**

35. Andrew has 12 marbles that are identical in size, but vary in color. Six are red, four are blue and two are green. If he wishes to place them in a straight line on a table, how many distinct arrangements can be made?

(A) 13,860 **(B) 119,760,795** **(C) 239,507,730** **(D) 359,254,665** **(E) 479,001,600**

36. Given: $\vec{v} = \langle 1, 2, 3 \rangle$ and $\vec{w} = \langle 4, 6, 8 \rangle$. The unit vector in the direction of $2\vec{v} + 3\vec{w}$ is the vector $\left\langle \frac{a}{d}, \frac{b}{d}, \frac{c}{d} \right\rangle$ where d = _____. (nearest hundredth)

(A) 35.35 (B) 36.45 (C) 37.55 (D) 38.65 (E) 39.75

37. Consider the hyperbola with equation $4y^2 - 9x^2 + 16y + 108x - 344 = 0$. The eccentricity of the hyperbola is _____. (nearest tenth)

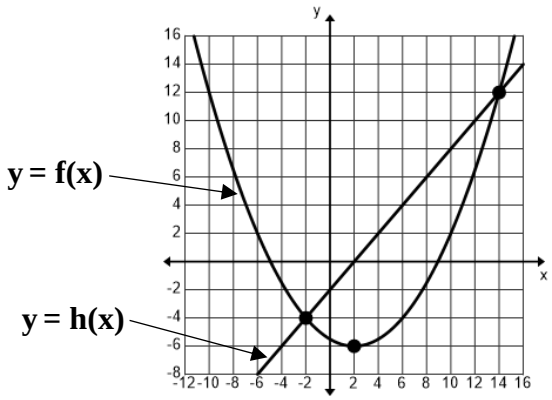
(A) 1.2 (B) 1.4 (C) 1.6 (D) 1.8 (E) 2.0

38. The "on base percentage" for Bobby Richardson of the Yankees is 0.328. If he has 9 at bats in a doubleheader against the Dodgers, what is the probability that he will safely get on base exactly 5 times. (nearest hundredth)

(A) 0.10 (B) 0.13 (C) 0.16 (D) 0.19 (E) 0.22

39. The graphs of $y = f(x)$ and $y = h(x)$ intersect at the points P and Q. PQ = _____. (nearest tenth)

(A) 21.7
(B) 22.0
(C) 22.3
(D) 22.6
(E) 22.9



Problems 39, 40, 41, 42

40. The point $F(2, b)$ is the focal point of the parabola. b = _____.

(A) –4.0 (B) –5.0 (C) –5.5 (D) –5.75 (E) –5.875

41. The area bounded by the graphs of $y = f(x)$ and $y = h(x)$ is _____. (nearest tenth)

(A) 84.7 (B) 85.0 (C) 85.3 (D) 85.6 (E) 85.9

42. If the area bounded by the graphs of $y = f(x)$ and $y = h(x)$ is revolved around the line $x = -6$, then the volume of the solid generated is _____. (nearest whole number)

(A) 6426 (B) 6430 (C) 6434 (D) 6438 (E) 6442

43. Consider the graph of $h(x) = 2\ln(x) - \frac{1}{e^x}$. The slope of the line tangent to the graph of h(x) at x = 9 is _____. (nearest thousandth)

(A) 0.218 (B) 0.222 (C) 0.226 (D) 0.230 (E) 0.234

44. Consider the graph of $2xy^2 - 3y + 4x^2 = 13$. The y-intercept of the line tangent to the curve at the point where y = 3 and $x > 0$ is _____. (nearest tenth)

(A) 5.3 (B) 5.5 (C) 5.7 (D) 5.9 (E) 6.1

45. Farmer Fred wants to make a rectangular holding area for his dairy cattle using 640 feet of fence. He plans to use the back side of his barn as one of the sides. The maximum possible value of the holding area is _____ square feet.

(A) 25,600 (B) 32,000 (C) 38,400 (D) 44,800 (E) 51,200

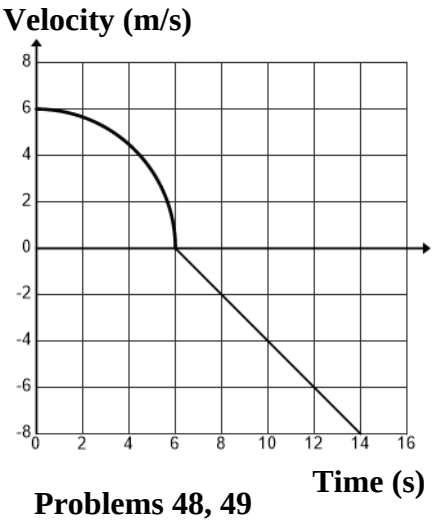
46. A 25-ft-long ladder rests against the wall of a building. The foot of the ladder begins to slide away from the building at a constant rate of 2 ft/s. How fast is the top of the ladder sliding down the wall at the instant the foot of the ladder is 7 feet from the wall? (nearest whole number)

(A) 7 in/s (B) 9 in/s (C) 11 in/s (D) 13 in/s (E) 15 in/s

47. The position of a particle is given by the parametric equations $x(t) = e^{.4t}$ and $y(t) = \ln(t^2 + 2)$ for $0 \le t \le 12$. Find the total distance traveled by the particle from t = 2 to t = 10. (nearest tenth)

(A) 51.5 (B) 51.8 (C) 52.1 (D) 52.4 (E) 52.7

48-49. The graph on the right consists of a quarter circle and a line segment. The graph represents the velocity of an object during a 14-second time interval.



Problems 48, 49

48. Find the object's average velocity during the 14-second time interval $[0, 14]$. (nearest hundredth)

(A) –0.33 m/s (B) –0.31 m/s (C) –0.29 m/s
(D) –0.27 m/s (E) –0.25 m/s

49. Find the object's acceleration at t = 10 s.

(A) -4.0 m/s^2 (B) -3.0 m/s^2 (C) -2.0 m/s^2
(D) -1.0 m/s^2 (E) 0

Time	11:00 AM	12:00 PM	2:00 PM	6:00 PM	8:00 PM
People/minute	6	9	7	11	5

50. Suppose that Larry's Cafeteria in Millersview opens their doors at 11:00 AM and closes their doors at 8:00 PM. The table above shows the rate at which people entered the cafeteria, in people per minute, at various times on Saturday. Use a trapezoidal approximation with four subintervals to estimate the total number of people who dined at Larry's on Saturday.

(A) 4122 (B) 4224 (C) 4326 (D) 4428 (E) 4530

51. The rate of change of a population of horned lizards at any time t, $t \geq 0$, is changing at a rate proportional to its population at time t. The population on March 1, 2000 was 180. On March 1, 2004 the population was 210. What should the population be on March 1, 2033?

(A) 642 (B) 644 (C) 646 (D) 648 (E) 650

52. Consider the curve given by $f(x) = x^3 + 6x^2 - 4x + 2$. The local maximum of f(x) is _____. (nearest tenth)

(A) 49.8 (B) 50.2 (C) 50.6 (D) 51.0 (E) 51.4

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Minutes	45	65	57	71	53	56	73

53-54. Joe tries to exercise every day at the gym. He runs, lifts weights, and uses a Stair Master. Last week, he recorded the time he spent at the gym as shown in the table above.

53. Find the positive difference between the mean and the median of the data.

(A) 0 min (B) 1 min (C) 2 min (D) 3 min (E) 4 min

54. A modified boxplot shows that there are _____ outliers.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Miles	35	48	65	72	86	100
Time	3 hr 22 min	3 hr 6 min	2 hr 55 min	2 hr 44 min	2 hr 30 min	2 hr 18 min

55-56. Six men of similar abilities spent 6 months preparing for the Houston Marathon. Their average weekly mileage and their times for the race are shown in the table above. Coach Salazar plotted the data in the table and decided that a linear relationship existed between the average weekly mileages of his runners and their times at the Houston Marathon. He used statistical software to generate a least squares regression line (LSRL).

55. The LSRL predicts that for each increase of one mile in a runner's weekly mileage, there is a corresponding decrease of _____ seconds in their marathon time. (nearest whole number)

(A) 56 (B) 59 (C) 62 (D) 65 (E) 68

56. According to the model, what should a runner's average weekly mileage be in order to run a marathon in 2 hr 10 min? (nearest whole number)

(A) 106 mi (B) 108 mi (C) 110 mi (D) 112 mi (E) 114 mi

57. In a random sample of 32 adult male wild turkeys found in Hemphill County, the average weight was 20 pounds with a standard deviation of 2 pounds. Construct a 96% confidence interval for the mean weight of adult male turkeys found in Hemphill County. (nearest hundredth)

(A) {19.36, 20.64} (B) {19.24, 20.76} (C) {19.12, 20.88} (D) {19.00, 21.00} (E) {18.88, 21.12}

58. At Aberdeen High School, 58% of the students are girls and 42% are boys. Suppose that 72% of the girls select soccer as their sport compared to 36% for the boys. If a randomly selected student selects soccer as his/her favorite sport, what is the probability that the student is a girl? (nearest hundredth)

(A) 0.65 **(B) 0.67** **(C) 0.69** **(D) 0.71** **(E) 0.73**

59-60. Assume that the average drive for a 74-year-old male golfer is 226 yards with a standard deviation of 12 yards.

59. If Randy is 74 years old and his average drive is 237 yards, what percentile does that place him at among 74-year-old male golfers?

(A) 78^{th} **(B) 80^{th}** **(C) 82^{nd}** **(D) 84^{th}** **(E) 86^{th}**

60. If a 74-year-old male golfer wanted to be at the 96^{th} percentile, what average drive is required? (nearest whole number)

(A) 241 yd **(B) 243 yd** **(C) 245 yd** **(D) 247 yd** **(E) 249 yd**

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University Interscholastic League
MATHEMATICS CONTEST
HS ● SAC ● 2024
Answer Key

1. D	**21. D**	**41. C**
2. A	**22. D**	**42. C**
3. B	**23. D**	**43. B**
4. E	**24. B**	**44. D**
5. C	**25. B**	**45. E**
6. D	**26. A**	**46. A**
7. A	**27. A**	**47. E**
8. A	**28. D**	**48. D**
9. C	**29. B**	**49. D**
10. A	**30. D**	**50. E**
11. B	**31. E**	**51. A**
12. E	**32. D**	**52. C**
13. C	**33. C**	**53. D**
14. D	**34. D**	**54. A**
15. A	**35. A**	**55. B**
16. B	**36. E**	**56. B**
17. E	**37. A**	**57. B**
18. E	**38. A**	**58. E**
19. B	**39. D**	**59. C**
20. C	**40. A**	**60. D**